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Journal of Sound and Vibration 271 (2004) 849–861

JOURNAL OF SOUND AND VIBRATION

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The measurement and prediction of sound waves of arbitrary amplitude in practical flow ducts

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Received 29 November 2002; accepted 3 March 2003

Abstract

A study is presented that explores the influence of the peak to mean pressure ratio on the wave action occurring in highly acoustically reactive ducts with significant flow present. Particular practical applications concern predictions of orifice noise emissions from piston engine or compressor intake and exhaust systems, with the effect of acoustic resonances on both excitation and acoustic power transmission. The results indicate that any practical differences between linear and non-linear predictions remain negligibly small when the pressure ratio remains below 1.1, corresponding to 170 dB spl, provided that the influence of all frequency-dependent physical features is included. Above this level of excitation in any lengths of uniform pipe connecting other system components, some new observations demonstrate the extent to which compression wave steepening may be of practical significance in the spectral distribution of the power propagated.

1. Introduction

It has long been recognized [1] that the propagation of one-dimensional progressive pressure waves involves non-linear processes to some extent. For example [1], those describing the simple compression and rarefaction waves induced respectively on the advancing and retreating sides of a piston moving in a uniform pipe with a steady or oscillatory motion. Such waves change their shape as they travel, since compressions become relatively steeper, while the intervals between zero (i.e., mean pressure) crossings remain constant. The rate of any such steepening depends on the wave strength [1,2], defined as the ratio of the pressure disturbance amplitude p_a to the mean

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doi:10.1016/S0022-460X(03)00756-9

pressure p_0, since the magnitude of the extreme fluctuations in the corresponding sound speed is expressed by

$$c_a/c_0 = (p_a/p_0)^{(\gamma-1)/2\gamma}, \tag{1}$$

where γ is the ratio of the specific heats of the medium. The practical significance of such steepening varies with each individual circumstance, such as the form of excitation with the resulting wave amplitude, together with the shape, extent and other relevant conditions at the confining and terminal boundaries of the pipe or duct.

Observations [3] of the fluid motion in a tube, open at one end and excited at resonance by an oscillating piston at the other, revealed that the flow near the mouth included flow separation with vortex shedding. A proportion of the energy of compression waves incident at such a pressure release surface is reflected as an expansion wave [2,3], with some energy dissipated and the remainder emitted. Similarly for expansion waves which reflect as compressions. The relative proportions of each part is frequency dependent, while the wave motion is no longer simple, but some combination of standing and progressive waves. Common examples of this include horns [4] and that existing [5–10] throughout cyclicly excited highly acoustically reactive intake and exhaust systems of piston engines and compressors. In all such cases, the equations [1] describing wave steepening in simple progressive waves no longer apply, while some frequency-dependent redistribution of fluctuating wave energy will occur at each area or other discontinuity along the system. As a first approximation [4–10], the local motion over a sequence of relatively short distances may be described as the linear combination of incident and reflected waves, with the modelling conveniently performed in the frequency domain. If necessary [4], this can be corrected after each step in the calculations for any significant influence of non-linearity.

Calculation [5–10] of the high amplitude wave action associated with piston engine and compressor breathing is commonly performed in time. The cylinders, valves and manifolds represent a time varying system while the remainder of the intake and exhaust [7–10] acts mainly as a passive wave filter. Non-linear wave propagation along uniform lengths of pipe may be described by Rieman invariants [5–10] and evaluated by the method of characteristics (MOC). Otherwise [8,10], numerical techniques that employ MacCormack, two-step Lax–Wendroff or similar shock capturing finite difference schemes might be preferred. However, the record shows that time domain descriptions of the wave action in those parts of the system with complex geometry may often fail to produce sufficiently realistic descriptions of wave transfer across elements or of orifice noise emissions.

1.1. Boundary conditions

Predictive modelling also requires the specification of appropriate boundary conditions at all sites along the flow path where wave reflection or generation takes place. Normally, the associated processes [11] are both frequency and flow dependent. During calculations in the time domain, this presents the problem of finding equivalent time varying descriptions of the corresponding reflection propagation and energy dissipation coefficients. Also the observed wave steepening with simple wave propagation in semi-infinite ducts differs significantly from that found [5,6] for continuously excited highly reactive systems. Furthermore, appropriate Fourier transformation [12] of linear frequency models [13] of sound reflection and radiation from open pipes was shown to yield acceptable predictions [7,14] of observed sound energy radiation to the surroundings, or of wave reflection, even with weak incident shocks [12] so long as an appropriate time delay [7,12] was included.

Similarly, with cyclic events, any results of measurements or calculations may be easily transferred [7–10] between time and frequency domains by appropriate Fourier or harmonic transformation. Thus any frequency-dependent boundary conditions can be represented by an appropriate transformation to the time domain, provided [7,8] the relevant time delays are also included. This suggests a hybrid approach [7,9,10] that takes advantage of well-established methods for calculating the cyclic thermodynamic and fluid dynamic processes in the cylinder, with the associated wave action in the manifold in the time domain [7,8], together with similarly well-validated methods [11] for calculating spectral descriptions of wave propagation in the remaining passive system elements. This requires appropriate interfacing [7] between the two domains. This also takes advantage of the fact that linear frequency domain calculations are commonly at least two orders of magnitude faster to compute than the corresponding ones in time.

2. One dimensional travelling pressure waves

Provided the transverse dimensions of the system elements remain a fraction of a wavelength, the resulting motion remains substantially one-dimensional over most of the flow path. The record [5,11,12] also shows that the wave motion rapidly becomes plane following area expansions and contractions. If viscous and other dissipative terms are neglected and also if shock waves and similar disturbances are absent, so long as the gas is homogeneous at some instant, the specific entropy s remains constant at all times. All quantities associated with such a sound wave propagated in the x direction depend on x and t only, while for a wave of any amplitude it may be assumed that the velocity v can be expressed as a function of density ρ, or alternatively of total pressure p. With entropy s constant, both density and sound speed c, where $c^2 = (\partial p/\partial \rho)_s$ are functions of total pressure p. The equation of continuity, or conservation of mass per unit volume, and Euler's equation, or Newton's second law of motion, are expressed respectively by

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial v}{\partial x} + v\frac{\partial \rho}{\partial x} = 0, \quad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0. \tag{2, 3}$$

Solutions of these exact non-linear equations for the general case of travelling combined with standing waves will be presented later.

2.1. The acoustic approximation

Assuming that the temporal fluctuations in pressure p' are always small in relation to their mean value p_0 so that $p = p_0 + p'$, similarly also that $\rho = \rho_0 + \rho'$, and $v = u_0 + u$, after substitution in Eqs. (2) and (3) followed by the neglect of all second order fluctuating quantities, the resulting two equations are linear to first order. They are then satisfied by [11]

$$p(x,f) = [p^+(0,f)\exp(-\mathrm{i}k^+x) + p^-(0,f)\exp(\mathrm{i}k^-x)]\exp(\mathrm{i}\omega t), \tag{4a}$$

$$\rho_0 c_0 u(x,f) = p^+(x,f) - p^-(x,f), \tag{4b}$$

where f is the frequency, $\omega = 2\pi f$, $p^\pm$ are component waves travelling respectively in the directions of positive and negative x, while $k^\pm$ are the corresponding wave numbers $\omega/(c_0 \pm u_0) = \omega/c_0(1 \pm M)$, where M is the mean flow Mach number u_0/c_0. In practical situations [11], when viscothermal losses are significant the wavenumbers are complex. Eq. (4a) normally represents [11] the superposition of a standing wave and a travelling wave. It also forms the basis [14] for decomposing the pressure signals from pairs of axially spaced flush mounted pressure transducers into $p^\pm(x,f)$ component wave spectra. The corresponding acoustic intensity, or energy flux per unit area [11,14], is then expressed by

$$I = \{|p^+|^2(1+M)^2 - |p^-|^2(1-M)^2\}/\rho_0 c_0. \tag{5}$$

with reactive systems, the energy flux is normally a small fraction of the total fluctuating energy.

2.2. Some practical examples of "linear" propagation

There have been a sequence of instances [14–19] where the observed power radiated from the open orifice to the surroundings has agreed closely with that predicted with the measured power flux in the adjacent pipe, based on linear analysis [14] with Eqs. (4) and (5). With more appropriate modelling that included the influence of mean flow Mach number and temperature [15,16], such agreement was found typically within ± 1 dB [14], between predicted and radiated spectral levels up to 1500 Hz. In this case the transducer spacing was 75 mm, maximum flow Mach number was 0.2 and peak spectral levels in the duct reached 160 dB, corresponding to extreme fluctuations in sound speed of less than 0.4 per cent. Similar good agreement with emitted power was found with spectral levels exceeding 165 dB in the exhaust tailpipe [17], where the power flux was determined using narrowband standing wave measurements obtained with a motorized axial traverse at flow temperatures above 500 K and Mach numbers reaching 0.17. Similarly, consistent agreement within ± 2 dB was found between emissions and predictions with tailpipe power flux measured [18] with a transducer spacing of 100 mm, spectral levels exceeding 150 dB, flow temperatures exceeding 670 K and Mach numbers up to 0.15. Finally, agreement within ± 3 dB between emitted and predicted power [19] was obtained when the power flux was measured with a transducer spacing of 180 mm, spectral levels approaching 170 dB, flow temperatures reaching 750 K and Mach numbers exceeding 0.2. With this last example, sound emission records were slightly contaminated by room effects, thereby increasing the scatter of the results.

From Eq. (1), the extreme fluctuations in sound speed in all these examples never exceeded 1 percent. Also, the transducer spacing was generally less than a quarter of the wavelength at the highest frequency of interest. Thus one would expect that under such conditions any practical differences between the measured behaviour and predictions based on linear models would hardly be obvious. However, fluctuating pressure levels approaching 190 dB are frequently observed in the manifolds of piston engines and compressors. The corresponding extreme fluctuations in sound speed would then approach 10 percent, so that significant differences may then be expected. A relevant practical example is presented in Section 3 below, having first described the application of the MOC to the solution of Eqs. (2) and (3).

2.3. Propagation of pressure waves of finite amplitude

Observations and theory [1] demonstrate that the wave shape alters significantly as it propagates when the wave amplitude is not small, so that [1,20] spectral descriptions of the wave motion may cease to be appropriate. With unsteady isentropic flow, one can replace the component waves $p^{\pm}(x,f)$, that describe the wave motion in the frequency domain, by small disturbances leaving a given point in the $x-t$ plane that are propagated in time along the three characteristics (C_+, C_-, C_0). The slopes dx/dt of $C_{\pm}$ are respectively $v+c$ and $v-c$, while C_0 moves with the gas flow. The equations of motion (2,3) can be combined [20,21] to give

$$\frac{\partial v}{\partial t} \pm \frac{1}{\rho c}\frac{\partial p}{\partial t} + \left(\frac{\partial v}{\partial x} \pm \frac{1}{\rho c}\frac{\partial p}{\partial x}\right)(v \pm c) = 0. \quad (6a, b)$$

Introducing the Rieman invariants $J_+ = v + \int dp/\rho c$ and $J_- = v - \int dp/\rho c$, while for a perfect gas one has

$$J_+ = v + 2c/(\gamma - 1), \quad J_- = v - 2c/(\gamma - 1), \quad (7a, b)$$

the equations of motion take the simple form [21],

$$[\partial/\partial t + (v + c)\partial/\partial x]J_+ = 0, \quad [\partial/\partial t + (v - c)\partial/\partial x]J_- = 0. \quad (8a, b)$$

The differential operators acting on J_+ and J_- are the operators of differentiation along the characteristics C_+ and C_- in the $x-t$ plane, while small perturbations of J_+ are propagated only along C_+ and those of J_- only along C_-.

With anisentropic flow, $dp/\rho c$ is not a perfect differential. Small perturbations of the form $\delta v \pm \delta p/\rho c$ are still propagated along the characterization of one family. Eqs. (6a,b) must be supplemented by the adiabatic equation

$$(\partial/\partial t + v\partial/\partial x)s = 0, \quad (6c)$$

which shows that the perturbations δ_s are propagated along the characteristic C_0 moving with the flow. An example follows of the application of characteristic solutions to the prediction of non-linear wave propagation in flow ducts in time [6], to be compared later with the results of linear solutions.

3. Measurement and prediction of "high amplitude" wave propagation

A quantitative description of the axial distribution of one-dimensional combined progressive and interfering wave motion, with the associated energy flux, requires [5,6,22,23] the establishment of simultaneous time histories of the associated fluctuating pressure and velocity over some reference plane along the duct. Simultaneous measurements of time-averaged temperature and pressure are also required. Furthermore, with flow present, the coherent signals associated with the wave motion must be distinguished [14,24] from all other signals associated with turbulence and other disturbed flow. It is also clearly desirable that any measurement procedures should not introduce further pressure fields that contaminate the signal records. Pairs of closely spaced flush mounted wall pressure transducers, combined with appropriate signal processing [6,14,19,22,24], can meet most of these requirements. Previously it was shown that [11,14,19,24], so long as fluctuating pressure levels were below 170 dB, linear processing based on Eq. (4) did produce close estimates of the spectral amplitudes of the corresponding positively and negatively travelling component pressure and velocity, with the associated power flux. Alternatively, MOC-based procedures [6,22] have also provided equivalent temporal records at much higher levels that approached 190 dB.

These waves [6,22,23] were generated in a straight tube 2.82 m long and 39 mm bore, supplied with air from a plenum maintained at 2.8 bar gauge pressure. The flow was interrupted by a rotating cylindrical valve that had a symmetrical convergent divergent passage giving choked flow at its throat. This produced two pulses per revolution with a peak to mean pressure amplitude ratio exceeding 1.6 at rates varying between 30 and 170 pulses per second. The most stable conditions [23] existed between 40 and 80 pulses per second, a range that included the first open pipe acoustic resonance lying close to 60 Hz. The pressure time histories from flush mounted National Semiconductor pressure transducers type LX 16046 B were recorded digitally with a parallel sample and hold system at 100,000/n samples per second, where n is the number of signal channels being acquired. Normally, this was 4, but sometimes 8 out of a possible 32. Four groups of tappings [6] were distributed axially along the tube providing sequences of transducer pair spacings from 50 to 200 mm with an extreme spacing of just over 1.3 m.

3.1. *Comparison of non-linear and linear predictions with measurements*

A pair of pressure transducers spaced 100 mm apart provided pressure time histories that were processed [6,22] to produce an estimated corresponding velocity record $v_r(t)$ at the reference plane situated 155 mm upstream of the midpoint of the pipe. Pressure records were also measured [6] at 625 and 1090 mm downstream of the reference plane for comparison with the predictions by the method of characteristics. These were made at 100 r.p.m. intervals between 1200 and 2400 r.p.m. or from 40 to 80 pulses per second. Some further sets of measurements and comparisons [6,23] were made both upstream and downstream of the reference plane. The results all show close agreement between measured pressure time histories and the MOC predictions [6,22] translated along the tube both upstream and downstream from the reference plane. These were all accompanied [6] by estimates of the velocity time histories, the surviving published records being tracings [6,23] of the graphical computer printouts.

A further surviving set of printouts for the valve rotating nominally at 1300 r.p.m., but actually at 1268 r.p.m., giving 42.26 pulses per second, was resampled at 128 equal time steps per rotation. The resulting pair of pressure time histories, $p_r(t)$ recorded at the reference plane and $p_\delta(t)$ at 100 mm downstream, are reproduced together in Fig. 1. After transformation with a standard fft routine, the resulting spectra were processed [14,19] to yield estimates of the corresponding component spectral amplitudes $p_r^{\pm}$ at the reference plane. Following translation 625 mm downstream and recombination with Eqs. (4) [7,11,14], the resulting spectral estimates of pressure $p(f)$ and velocity $v(f)$ were transformed with an inverse fft to the corresponding time history estimates $p(t)$ and $v(t)$ at this plane. The resulting estimates are compared with the measured pressure in Fig. 2a and the MOC-derived estimate of velocity in Fig. 2b. Although there is a fair match in both cases, there are some indications of wave steepening in the measured pressure waveform compared with the corresponding linear estimate.

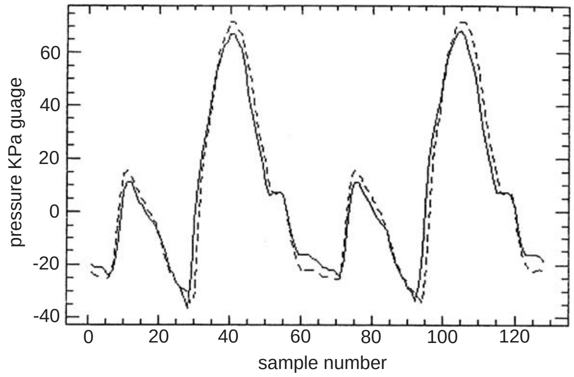

Fig. 1. Pressure time histories resampled: ——, at reference plane; – – – – –, at 100 mm downstream.

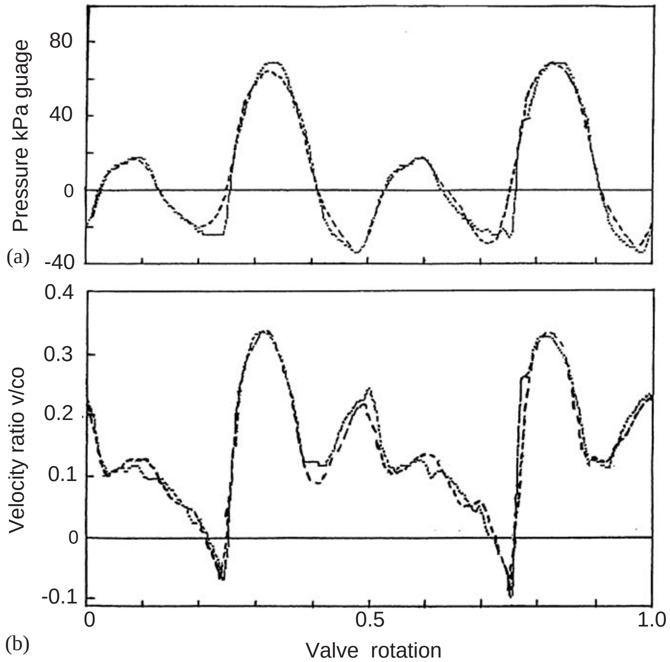

Fig. 2. Time histories, 625 mm from reference plane. (a) Pressure: ——, measured; – – – – –, linear transfer. (b) Velocity: ——, estimated MOC; – – – – –, predicted linear.

3.2. Wave steepening

A clearer demonstration of this is provided by a comparison of the corresponding pressure spectra. That measured at the reference plane is compared with the linear prediction at 625 mm in Fig. 3a. The general similarity in the overall trends of the spectral amplitude distribution suggests that any detailed differences are mainly due to wave interference but may also include some calculation errors. In contrast to this, the systematic difference between measured and linear

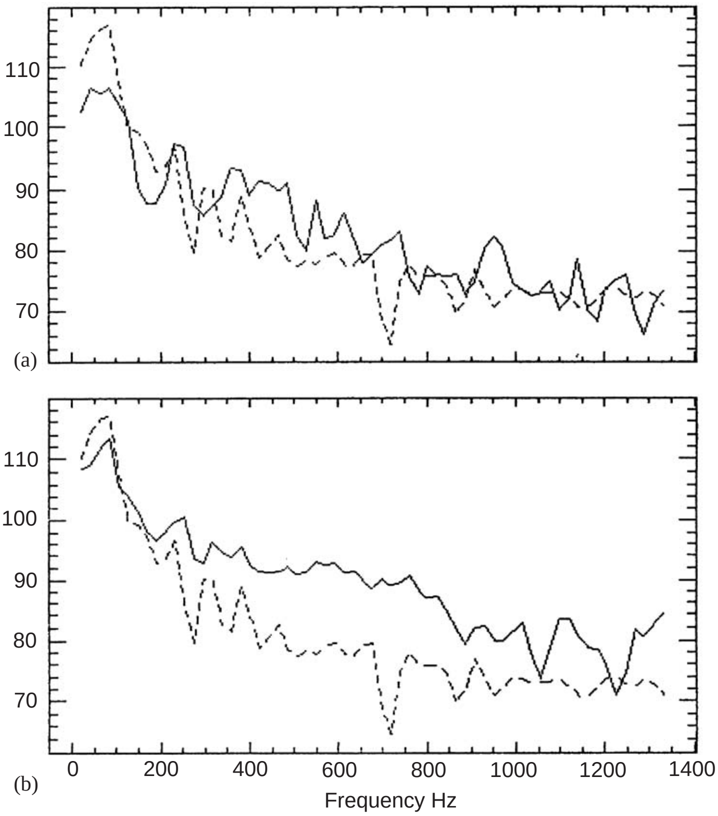

Fig. 3. Relative spectral levels, $20\log[p(f)/p_{\mathrm{ref}}]$. (a) ——, at reference plane; – – – – –, linear transfer to 625 mm. (b) ——, non-linear transfer; – – – – –, linear transfer to 625 mm.

spectral estimates at 625 mm in Fig. 3b give clear indications of the systematic transfer of wave energy from lower to higher frequencies in the measurements, that is absent from the linear estimates. The presence of wave steepening is also illustrated by the triple comparisons between the observed, the non-linear estimates by MOC [6] and new linear predictions of the pressure time histories in Fig. 4a at 625 mm and 4b at 1090 mm respectively. Clearly the MOC estimates give a closer match with the measured waveform profile than do the linear ones.

It has been noted earlier [23] that the non-linear predictions of pressure time histories were normally in agreement with observation [6] and that both the cycle averaged, predicted and measured wave motion satisfied conservation of mass, of energy and of momentum in all these experiments. These facts all support the conclusion that the measurements with the related predictions or estimates were all in close agreement with reality. Furthermore, there is a striking similarity between the observations [6,23] and expected reactive acoustic behaviour, as one might expect, although some systematic differences exist due to wave steepening. A further illustration of this is provided by the comparisons in Fig. 5 between linearly and non-linearly derived estimates of the cyclic velocity fluctuations at the inlet and after propagation to the outlet end of the pipe, that resulted from excitation at the first open pipe acoustic resonance. The relative wave

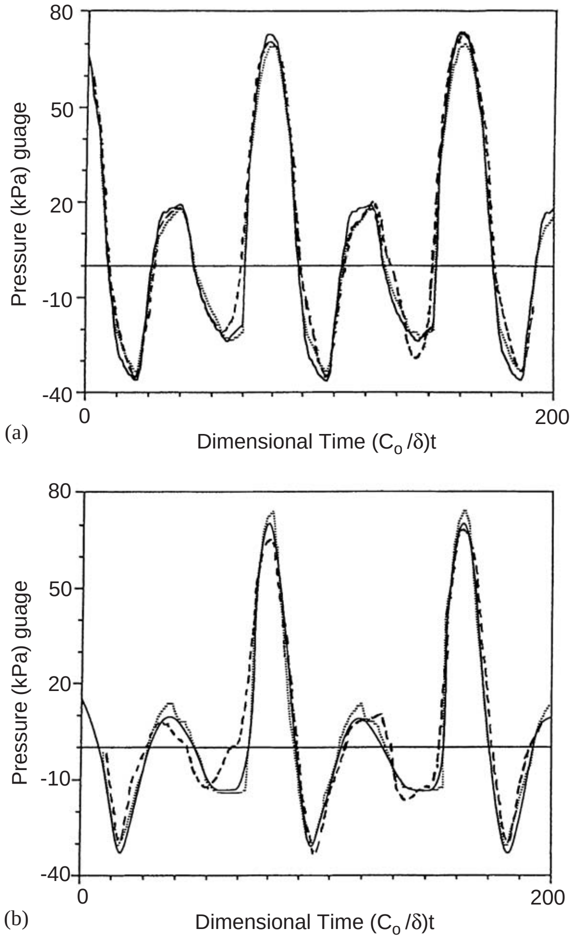

Fig. 4. Comparison of measured with estimated pressures (a) 625 mm, (b) 1090 mm downstream from reference plane. ——, measured; ······, estimated non-linear; – – – – –, estimated linear.

steepening is demonstrated in the similarity of the waveforms at the inlet in Fig. 5a in contrast to the relative distortion at the outlet in Fig. 5b. Conservation of mass flow is also clearly demonstrated by the similarity in areas under the cyclic velocity records in both Figs. 5a and b. This is made more obvious by the comparison in Fig. 6 of the linear estimate at the outlet in Fig. 5b with that of the non-linear estimate at the inlet in Fig. 5a.

3.3. Estimates of velocity time histories

The similarity in the velocity waveform at a reference plane that was estimated either with linear analysis or non-linear MOC-based methods [6,22,25] is hardly surprising, since the flush mounted pressure transducers were closely spaced and always only a fraction of a half wavelength apart, even at the highest frequency of interest. Both these procedures require values of the mean cyclic

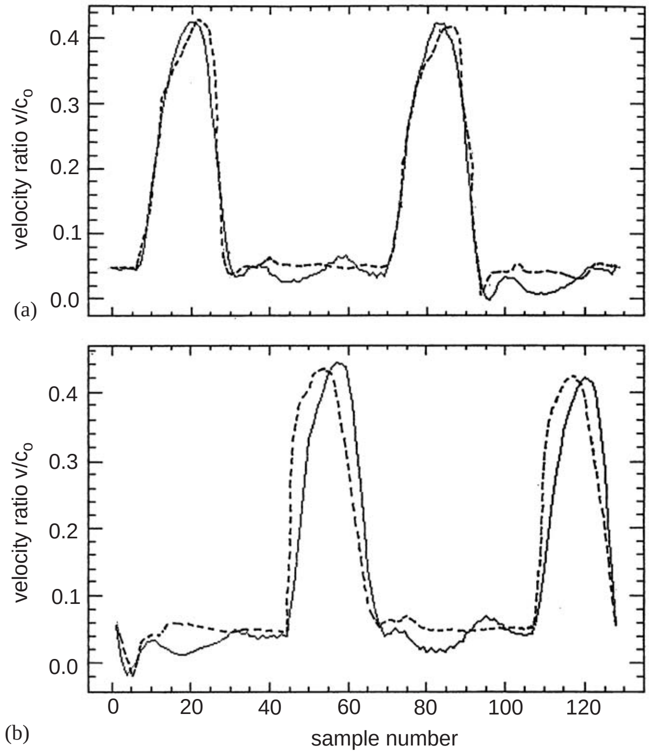

Fig. 5. Estimated velocity time histories at resonance (a) at inlet, (b) at outlet. ——, linear; – – – – –, MOC.

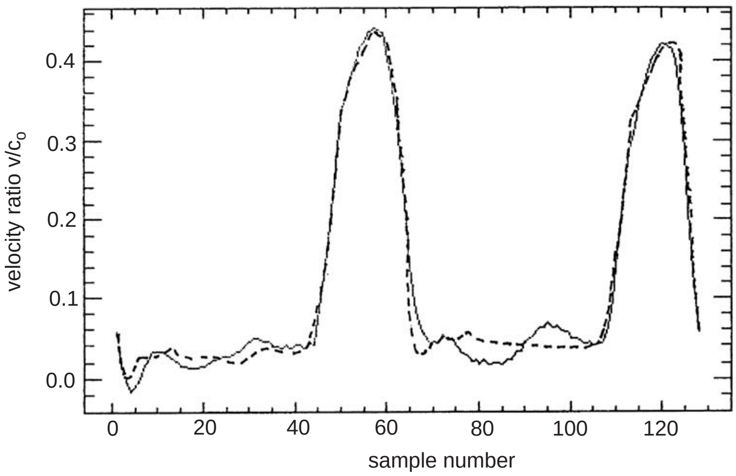

Fig. 6. Estimated velocity time histories at resonance: ——, linear at outlet; – – – – –, MOC at inlet.

mass flow, temperature and pressure as well as a pair of pressure time histories $p_r(t)$ and $p_\delta(t)$. The non-linear MOC approach also involves computer-automated iterative procedures [22] working with the two cyclic pressure records on the $x-t$ plane to establish equivalence between a sequence of initial velocities $v(t_1)$ and the calculated value $v(t_1 + T)$, a period T later. Each coincidence then provides a possible appropriate value $v_a(t_1)$. Also, the corresponding cycle averaged velocity $\langle v(t) \rangle$ for each selected initial value of $v_a(t)$ was calculated. The result that corresponds closely to the actual mass flow then establishes the most likely value of $v_a(t)$ for calculating a sequence of appropriate velocity estimates $v_1 \ldots v_n$ throughout the cycle at the reference plane. However, these estimates of the cyclic velocity time history $v_r(t)$ are restricted [22] to the somewhat limited sequence of calculated values at the corresponding times.

An alternative estimate that can normally provide an equivalent cyclic time history $v_r(t)$ with a significantly superior time resolution can be obtained by the well-established spectral methods based on Eqs. (4) [14–19,24], as is shown by the comparisons, in Figs. 5 and 6. Former studies [24,25] combined with sets of new comparisons with the data in Ref. [6], some of which have been summarized here, all confirm that a close coincidence normally exists between the MOC or equivalent time domain estimates and the transformed frequency domain estimates of the velocity time history $v_r(t)$ at the reference plane. With spectral levels exceeding 165 dB say, more realistic predictions of sound propagation along flow ducts should be provided by a hybrid approach that combines such potentially higher resolution velocity time history estimates at the reference plane, with MOC or equivalent numerical techniques [8] to transfer the wave motion elsewhere along the duct.

4. Discussion and conclusions

Although all acoustic wave motion involves non-linear processes to some extent, experience demonstrates that their practical influence depends on each individual circumstance. Thus the wave steepening that accompanies simple wave propagation [1] differs significantly [4–19] from that observed in the lengths of uniform pipe connecting other component elements of highly acoustically reactive flow duct systems, such as intakes and exhausts. One reason for this is that in such cases, the wave motion normally consists of a combination of progressive with standing waves, with the propagating wave energy remaining a rather modest fraction [14,17–19] of the total fluctuating energy. This difference is also associated with significant spectral energy redistribution [3,5,11,14] taking place at area and other discontinuities along the flow path, since both the wave transfer and reflection processes that occur there are normally strongly frequency dependent. Thus any progressive axial changes in waveform during propagation throughout complete systems are influenced by several frequency-dependent factors including wave interference and viscothermal attenuation, as well as non-linear amplitude-dependent steepening of compression waves. Similar considerations [4,8,26–28] apply to ducts with axially varying cross-section.

One-dimensional isentropic wave action and wave energy propagation along uniform pipes continuously excited at any pressure amplitude is demonstrated by Eqs. (2) and (3), or alternatively by Eqs. (6a,b). Realistic predictions calculated with MOC or equivalent numerical methods require simultaneous time history records of total pressure and velocity, or mass flow, at some reference plane, accompanied by all other relevant physical conditions. Similarly, these are also required for predictions with the linearized acoustic Eqs. (4a,b) when they are appropriate. In all cases the calculations require adequate specification of the associated time or frequency-dependent boundary conditions at every site along the flow path where wave reflections occur.

The failure of several existing codes to calculate sufficiently realistic predictions of orifice noise emissions by piston engines and compressors frequently arises [7–19,27–29] from deficiencies in the boundary conditions that were adopted. Otherwise, experimental evidence presented above confirms that close estimates to the required velocity time histories can be derived with the signal records from closely spaced pairs of flush wall mounted pressure transducers. The measurements, together with earlier results [6–10], also show that more realistic predictions of the wave action in uniform pipes with pressure amplitudes above 170 dB, say, result from appropriate non-linear calculations in time rather than the linear equivalent in the frequency domain. Note however that calculations by the mesh method of characteristics [28] may require correction [8,29] before the results satisfy conservation of mass or momentum.

The evidence presented here, or in the references cited, supports the conclusion that, with peak to mean pressure amplitude ratios of less than 1.1 or thereabouts in cyclicly excited acoustically reactive flow ducts, any difference between non-linear and linear predictions of the wave action in short ducts should remain negligibly small in practical terms. Provided due account is included for the presence of standing waves, these predictions also correspond closely to the measurements. The evidence also shows how close the coincidence between predicted and measured waveforms must be, for example, before realistic orifice noise emissions from piston engine or compressor intakes and exhausts can be calculated. In particular, such estimates based on single pressure records measured or predicted at one plane alone, without the corresponding velocity time history record, must generally remain inadequate in this context. Such deficiencies may also explain the discrepancies often reported between measured and predicted orifice noise emissions, where the full influence of the presence of standing waves with other frequency-dependent factors has been neglected. Finally, the result of wave interference is clearly illustrated by the detailed differences in the spectral distributions shown in Fig. 3a and by the observed influence [4–11,14–19,22–29] of acoustic resonance on both the excitation and the resulting fluctuating pressure distribution in practical flow ducts.

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